2D Modeling of Litz Wire
Reduced Model and its Application for Switched Reluctance Machine

M. Al Eit, F. Bouillault, C. Marchand, G. Krebs and L. Santandrea

GeePs, C.N.R.S UMR 8507, CentraleSupélec, UPSud and UPMC
11, rue Joliot Curie, Plateau de Moulon, 91192 Gif-sur-Yvette, France, moustafa.aleit@supelec.fr

The strong correlation between the level of eddy current losses and the winding geometry shows all the necessity of special attention to the manner of disposition of coils in machine slots and to the wire type whether it's solid or stranded conductor. Since the transposition of winding strands is a recommended solution to reduce eddy current losses, this article suggests an electromagnetic analysis of complex stranded conductors such as litz wire (LW). It is based on a 2D finite element (FE) model. A model reduction is also proposed. It allows benefiting from only one complete FE solution to find fast solutions in the slots domains when any variation of geometrical or physical data occurred. It allows different problem adapted meshes. Seeing that it prevents the Newton-Raphson iterations, the reduction model presents clear interests in repetitive analysis such as winding optimization processes.

***Index Terms*— Finite element analysis, eddy currents, Litz wire, model reduction, switched reluctance machine.**

I. INTRODUCTION

The copper losses are subdivided into classical ohmic DC losses and additional eddy current losses. The latter exists due to the strong electromagnetic coupling between the current density and the time varying magnetic fields penetrating the copper conductors. Due to the fact that this interaction between electric and magnetic variables cannot be solved easily, then FE methods can be used to give a numerical solution. The LW reduces eddy currents in an effective way because of the strands transposition. Hence, the 3D FE model is the suitable solution to pick up the electromagnetic effects. In spite of its precise solution, the 3D resolution leads to a substantial calculation time and requires large storage capacity. These heavy constraints hinder any process of optimization of winding geometries in terms of copper losses. Therefore, this article proposes a 2D FE model of the LW. Also it suggests a model reduction that provides clear advantages in repetitive analyses for optimization problems when we search the optimized geometry distribution and the optimized achievable number of strands for a given machine winding. The test problem is a switched reluctance machine (SRM) wounded with solid or LW conductors. The SRM can be used in hybrid or electric vehicle where, for autonomy considerations, energy efficiency is crucial [1].

II. FE METHOD FORMULATION

At low frequencies, the Maxwell's equations in electromagnetic systems are given by:

$$\overrightarrow{rot}(\vec{H}) = \vec{J},\ div(\vec{B}) = 0,\ \overrightarrow{rot}(\vec{E}) = -\frac{\partial \vec{B}}{\partial t} \qquad \text{(1a-b-c)}$$

The associated constitutive medium relationships are:

$$\vec{H} = \nu\vec{B},\ \vec{J} = \sigma\vec{E} \qquad \text{(2a-b)}$$

An easy coupling with the electric circuits leads to use in a privileged way the potentials for example the magnetic vector potential $\vec{A}$ and the electric scalar potential *V.* The resultant equation from (1) and (2) is given by:

$$\overrightarrow{rot}\left(\nu\overrightarrow{rot}(\vec{A})\right) = -\sigma\frac{\partial \vec{A}}{\partial t} - \sigma\overrightarrow{grad}(V) \qquad (3)$$

The variational formulation of (3) through machine cross section with the FE method as well as taking into account the circuit equation reflecting that the current feeding the conductor is *I(t)* lead to the following matrix system [2]:

$$\begin{pmatrix} [S]+\frac{[T]}{\Delta t} & [D] \\ [D]^t & \Delta t[G] \end{pmatrix} \begin{bmatrix} [A_t] \\ [\Delta V] \end{bmatrix} = \begin{bmatrix} \frac{[T]}{\Delta t}[A_{t-\Delta t}] \\ \Delta t[I(t)] + [D]^t[A_{t-\Delta t}] \end{bmatrix} \qquad (4)$$

ΔV is the electric voltage drop per unit length. Moving band technique is used to perform the rotor motion and Newton-Raphson iterations are adopted to take into consideration the non linearity of the magnetic circuits.

III. 2D MODELING OF LW

We consider the case of a conductor, placed in the slot of an electric machine, subdivided into *p* parallel strands and fed by the imposed current *I(t).* The individual strand currents can be different as long as their sum is equal to the total current *I(t)* [3]. This asymmetric repartition of the current depends on the time varying magnetic fields penetrating the conductor in the winding slot; it's then a function of the strands positions relative to the magnetic circuit and the air gap. In a given point in the slot, the magnetic field varies either due to the time varying current (skin and proximity effects) or to the stray flux caused by the salient rotor motion [3]. The strands of the LW switch their positions in the slot throughout the length of the machine. If the influence of the end windings is not considered, then the transposed strands will be subjected globally to the same electromagnetic effects. Taking into account this fact can be apprehended in a 2D FE model by enforcing the same current *I(t)/p* in each of the transposed strands. The assumptions that the strands in the LW case are in parallel and fed by the same current are added to system (4) in order to obtain a FE solution of the problem.

IV. MODEL REDUCTION

A SRM 8/6 is taken as an example. Half of the machine is studied for periodic reasons. Only one phase is fed, it consists of two coils connected in series; each of them has *n* turns. These turns can be distributed in the slot in different ways.

Each turn is a LW of surface S; it's composed out of a variable number of transposed strands p with multiple geometry partitions. In order to prevent for each winding configuration a non linear dynamic Complete FE (CFE) model, only one non linear static CFE model of the machine is performed. The coil is supposed to consist of 1 turn. The solid conductor of surface nS is fed by the position dependent current $nI(t)$ (Fig. 1). At each position the current density is uniform across the conductor cross section, hence the static nomination.

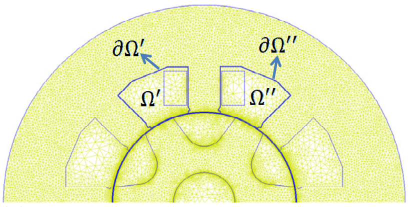

Fig. 1.CFE model

Once the static CFE model is solved, we pick up the position dependent vector potential solutions on the nodes of the boundaries $\partial\Omega'$ and $\partial\Omega''$ (Fig. 1).We apply them as boundary conditions to the Reduced FE (RFE) problems in the subdomains Ω_1 and Ω_2 (Fig. 2) respectively where the real winding configuration of conductivity σ is considered [4].

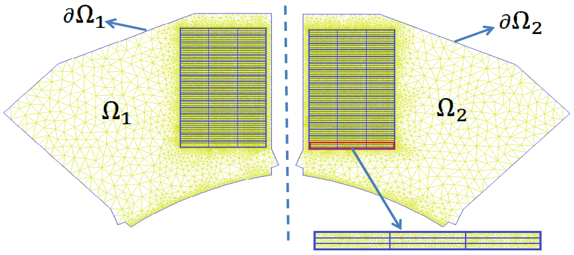

Fig. 2. RFE models, Left: forth conductor of the coil. Right: back conductors of the same coil.

Since the back and forth conductors are subjected to different magnetic stray fields, their slots domains are studied separately. Fig.2 shows an example of 18 turns placed horizontally. Each turn is a LW which is composed out of 9 strands (3×3) and fed by the current $I(t)$. It's about two non moving linear dynamic problems where the current density vector in the conductor material is governed by the equation:

$$\vec{J} = -\sigma\frac{\partial\vec{A}}{\partial t} - \sigma\overrightarrow{grad}(V) \qquad (5)$$

The subdomains Ω' and Ω_1 may be meshed with distinct densities; for that purpose, the mortar element method is used to determine the vector potential at the nodes of the boundary $\partial\Omega_1$ as a function of the boundary $\partial\Omega'$ [5] (same for Ω'' and Ω_2).

V.APPLICATION EXAMPLE

The SRM 8/6 is wounded with 18 vertical and horizontal turns as two examples and fed by the current $I(t)$ [1] (Fig. 3). The comparison between solid and LW turns shows the advantage of such wires in reducing the eddy current losses (TABLE1, Fig. 3) and their ranges of variation (Fig. 4). The RFE model insures in case of LW turns an efficient and accurate computation of eddy currents effects (TABLE1, Fig. 3, and Fig. 4). The calculation time is divided by a factor of 20.

TABLE I
MEAN COPPER LOSSES IN CASE OF SOLID TURNS (DYNAMIC CFE SOLUTION) AND LW TURNS (DYNAMIC CFE AND RFE SOLUTIONS)

	Copper losses (W)			**Ohmic DC losses (W)**
	Dynamic CFE model		**RFE model**	
	Solid conductor	**LW**	**LW**	
18 horizontal turns	180,22	143,46	147,16	120,2
18 vertical turns	273,49	149,79	152,59	

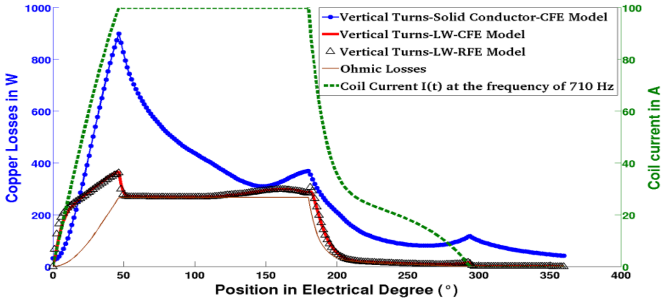

Fig. 3.Copper losses in case of solid vertical turns (dynamic CFE solution) and LW vertical turns (dynamic CFE and RFE solutions)

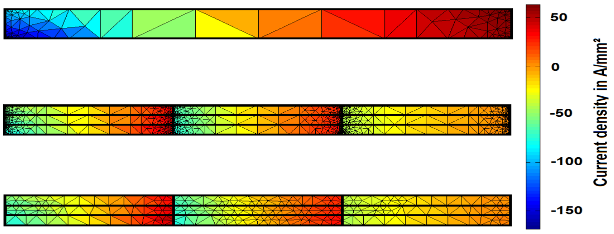

Fig. 4. Current density in the solid horizontal back conductor close to the air gap with dynamic CFE solution (up), LW with dynamic CFE solution (middle), LW with the RFE solution (down) (Position=46,5°).

VI. CONCLUSION

The 2D FE model of LW allows verifying the interest of this configuration to reduce eddy current losses. The reduction model, preventing time consuming non linearity iterations and the moving band technique, allows precise and fast calculation of copper losses and current densities in such wires individually. It can be used in winding conception processes for applications of severe energy constraints such as the SRM.

VII. REFERENCES

[1] H.Hannoun *et al.*, "Experimental validation of a switched reluctance machine operating in continuous-conduction mode," IEEE Trans. Veh. Technol., vol. 60, no. 4, pp. 1453-1460, May 2011.

[2] F. Piriou and A. Razek, "Coupling of saturated electromagnetic systems to non linear power electronic device," IEEE Trans. Magn., vol. 24, no. 1, pp. 274-277, Jan. 1988.

[3] C. Carstensen, "Eddy Currents in Windings of Switched Reluctance Machines," Ph.D, Univ. Rhénanie Westphalie, Germany, Nov 2007.

[4] P. Dular *et al.*, "Perturbation finite element method for magnetic model refinement of air gaps and leakage fluxes," IEEE Trans. Magn., vol. 45, no. 3, pp. 1400–1403, Mar. 2009.

[5] F. Rapetti *et al.*, "Calculation of eddy currents with edge elements on non matching grids in moving structures," IEEE Trans. Magn., vol. 36, no.4, pp. 1351–1355, Jul. 2000.